AXISYMMETRIC THERMAL PLUMES

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GRADUATE MODELLING CAMP

AXISYMMETRIC THERMAL PLUMES

RISING FLUID DUE TO A POINT SOURCE OF HEAT

HEATED FLUID LESS DENSE THAN SURROUNDINGS

CAN OCCUR IN: ATMOSPHERE
OCEAN
MANTLE OF EARTH

EXAMPLES

- URBAN THERMAL PLUME

URBAN AREAS WARMER THAN SURROUNDING AREA
(HEAT ABSORBED BY BUILDINGS, ROADS)

URBAN HEAT ISLAND

- INDUSTRIAL POWER PLANTS

  EXHAUST GASES FROM TURBINES

  COOLING TOWERS

- GEYSERS

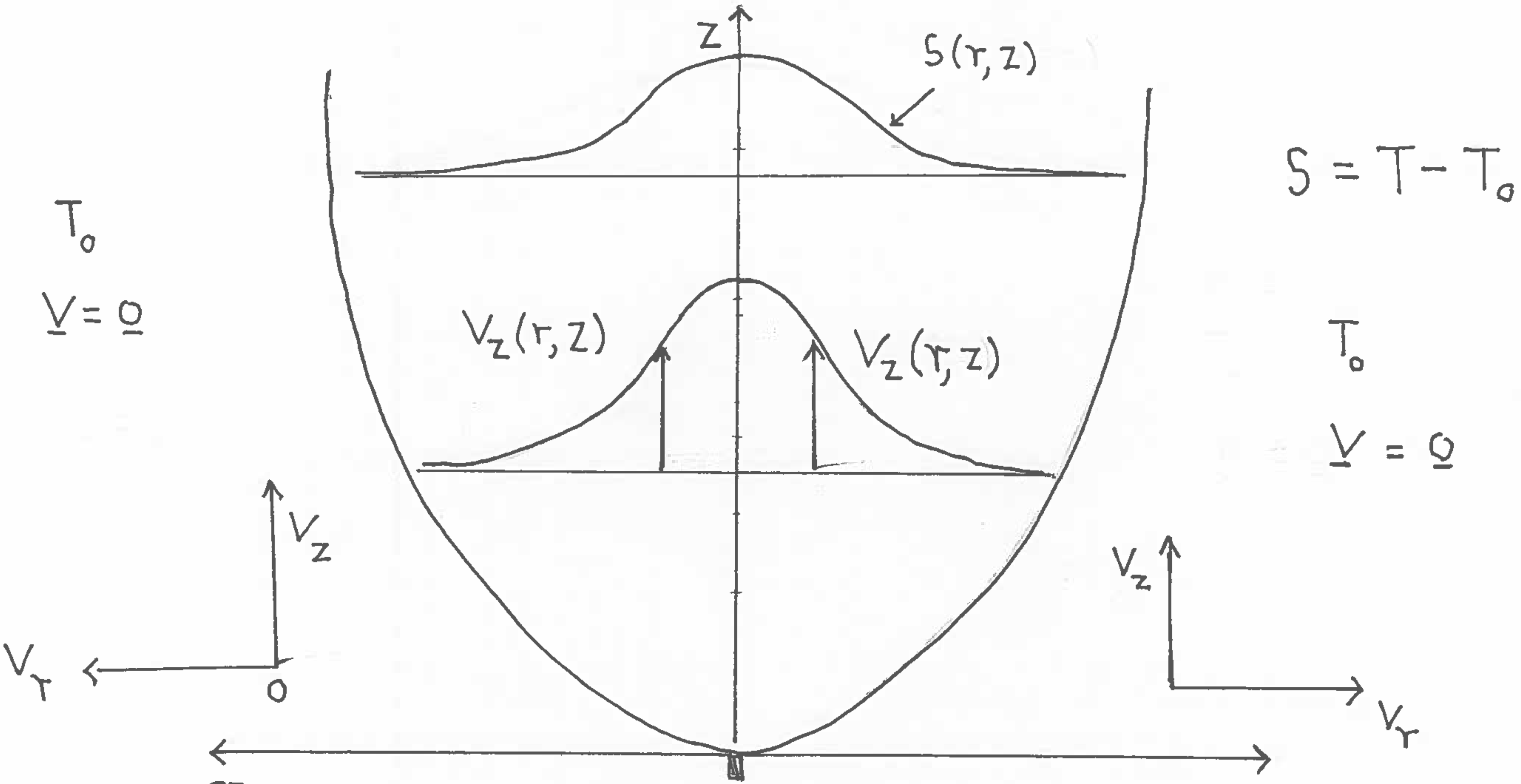
  HOT WATER AND STEAM RISING FROM UNDERGROUND HEAT SOURCE

- MANTLE PLUMES

  UPWELLING OF MOLTEN ROCK FROM HEAT SOURCES DEEP IN THE EARTH'S MANTLE

MATHEMATICAL MODEL

AXISYMMETRIC THERMAL PLUME



POINT THERMAL ENERGY SOURCE

BOUSSINESQ APPROXIMATION:

DENSITY CONSTANT EVERYWHERE EXCEPT IN BUOYANCY TERM

- NAVIER-STOKES EQUATION $\left(\frac{\partial}{\partial t}=0\right)$

$$z: \quad \rho\left[v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z}\right] = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2}\right] - \rho g$$

BUOYANCY TERM

$$r: \quad \rho\left[v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z}\right] = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2}\right]$$

- CONSERVATION OF MASS

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0$$

- ENERGY EQUATION

$$v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z} = \frac{k}{\rho c_v}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right]$$

- DEVIATION FROM BACKGROUND STATE

- DIMENSIONLESS VARIABLES

REYNOLDS NUMBER $Re = \dfrac{\text{INERTIA TERM}}{\text{VISCOUS TERM}}$

GRASHOF NUMBER $Gr = \dfrac{\text{BUOYANCY FORCE}}{\text{VISCOUS FORCE}}$

PRANDTL NUMBER $Pr = \dfrac{\text{VISCOUS DIFFUSION}}{\text{THERMAL DIFFUSION}}$

- BOUNDARY LAYER APPROXIMATION

NEGLECT TERMS $O\left(\frac{1}{Re}\right)$

$$V_r \frac{\partial V_z}{\partial r} + V_z \frac{\partial V_z}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r}\left[r \frac{\partial V_z}{\partial r} \right] + S$$

BUOYANCY TERM

$$\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z} = 0$$

$$V_r \frac{\partial S}{\partial r} + V_z \frac{\partial S}{\partial z} = \frac{1}{Pr} \frac{1}{r} \frac{\partial}{\partial r}\left[r \frac{\partial S}{\partial r} \right]$$

- CONSERVED QUANTITIES
- SIMILARITY TRANSFORMATION

  REDUCTION OF PDEs TO ODEs
- ANALYTICAL SOLUTIONS FOR SPECIAL Pr

  (PART PLAYED BY BUOYANCY TERM)

- APPLICATION TO REAL PLUMES

INVESTIGATE IF ANALYTICAL SOLUTIONS

YIELD NEW RESULTS

SUMMARY

MODELLING: BOUSSINESQ APPROXIMATION

BOUNDARY LAYER APPROXIMATION

Re Gr Pr

SOLUTION: EXACT SOLUTIONS

PART PLAYED BY BUOYANCY TERM

ANALYSIS OF RESULTS: APPLICATION TO REAL PLUMES